Ngā Whetū Resources

Physics Year 13 (NCEA Level 3)

The circular satellite orbit

Newton's law of gravitation: $F_g = G\frac{Mm}{r^2}$ *M* is mass of the earth; *m* is mass of the satellite; *r* is the distance between satellite and centre of the earth; *G* is the gravitational constant.

Centripetal force: $F_c = \frac{mv^2}{r}$ *m* is mass of the satellite; *v* is the linear speed of the satellite; *r* is the radius of the circle.

In a circular satellite orbit these forces are equal: gravitation supplies the centripetal force to maintain the circular motion.
Another way of looking at this is that the satellite is in a continuous *free fall* motion around the earth. No other forces are acting so there is an equilibrium (continuous motion).

There is a general mis-conception that there is no gravity outside the atmosphere or at a certain height above the earth and that this explains why astronauts in the space shuttle are weightless. This is of course nonsense. Look at Newton's law of gravitation. Only when *r* is infinite the gravitational force is zero.

The reason for weightlessness is that the shutlle (and the astronauts) are in a constant free fall motion. There are no other forces acting. Therefore there is zero-gravity in that situation. A spacecraft on an interplanetairy mission will also be in a free fall motion as soon as the engine is turned off. In space travel rocket engines are only used to come into a particular orbit or to change orbit. All other motion is free fall, just like all celestial objects in the universe are continuously moving under the influence of gravity.

The consequence of the two forces being equal is that:

$$\frac{mv^2}{r} = G\frac{Mm}{r^2} \quad \text{or} \quad \boxed{v^2 = G\frac{M}{r}} \qquad (1)$$

Thus $$v = \sqrt{G\frac{M}{r}}$$

Note that the mass of the satellite has disappeared and is thus not relevant[1]

[1] *If an astronout in the huge Space Station releases a pen it appears to be weightless because the pen is in the same orbit as the Space Station itself (and everything else in it)*

The time period for circular motion is $T = \frac{2\pi r}{v}$ or $\boxed{v = \frac{2\pi r}{T}}$ (2)

Combining (1) and (2) gives: $G\frac{M}{r} = \frac{(2\pi)^2 r^2}{T^2}$ or $\boxed{GM = \frac{(2\pi)^2 r^3}{T^2}}$ (3)

We can re-arrange (3) in three different ways:

A) $\frac{r^3}{T^2} = \frac{GM}{(2\pi)^2} = \text{constant}$ for a given central body, e.g. the earth. This is **Kepler's 3rd law**.

B) $r = \sqrt[3]{GM\left(\frac{T}{2\pi}\right)^2}$ This formula gives the distance *r* as a function of the period *T*.

C) $T = \sqrt{\frac{(2\pi)^2 r^3}{GM}}$ This formula gives the period *T* as a function of distance *r*.

Remember that M is the mass of the central (gravitating) body. For a satellite orbiting the earth M is the mass of the earth; for a satellite of e.g. Jupiter, M is the mass of Jupiter.

Problems to illustrate the use of these formula's.

Use the following constants:

G	6.67x10-11 $m^3kg^{-1}s^{-2}$
M_{earth}	5.98x1024 kg
M_{sun}	1.99x1030 kg
Radius Earth	6.37x106 m
distance Earth – Moon	3.84x108 m (centre to centre)
distance Earth – Sun	1.49×10^{11} m (centre to centre)

- Use *B)* to calculate the radius of a geostationary satellite (*T* = 24 hours). What is the height of the orbit above the earth's surface? *(Answer: H = 35,900 km)*
- Use *C)* to calculate the period of a satellite at a distance of 1000 km above the surface of the earth (do not forget to add the radius of the earth!). *(Answer: T = 1h 45 min)*
- Use *C)* to calculate the period (in days) of the moon. Does your answer make sense? *(Answer: T = 27.4 days)*
- Use *C)* to calculate the period (in days) of the earth about the sun. Does that make sense? *(Answer: T = 363 days)*

Note:
Much more about orbits in our Ebook "Falling in Space". See website.