Physics Year 13 (NCEA Level 3)

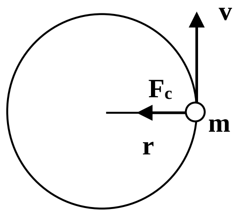
Summary Circular Motion

Frequency and Period

Period T is the time for one revolution (unit s)
Frequency f is the number of revolutions per second (unit s^{-1})

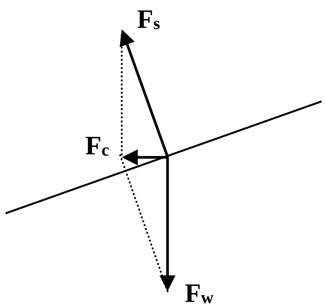
Period and frequency are each other's inverse: $T = \frac{1}{f}$

Centripetal Force and Acceleration



Uniform circular motion needs a constant centripetal acceleration (directed towards the centre) $a_c = \frac{v^2}{r}$ where v is tangential linear velocity and r is the radius.

Therefore the centripetal force (with $F = ma$) is $F_c = \frac{mv^2}{r}$



Example: car driving in a banked icy bend (no friction). The sum of support force (F_s) and weight force (F_w) forms the required centripetal force (F_c)

Satellite in Circular Orbit

Centripetal force is delivered by force of gravity:

$F = G\frac{mM}{r^2} = \frac{mv^2}{r}$ hence $v^2 = G\frac{M}{r}$; (M is Earth's mass, m is satellite's mass)

(Note that v^2 is now independent of the satellite's mass).

Period of the satellite: $T = \frac{2\pi r}{v}$ (circumference divided by speed).

Combine the last two equations and solve for r:

$$r = G\frac{M}{v^2} = G\frac{MT^2}{4\pi^2 r^2} \text{ hence } r^3 = G\frac{MT^2}{4\pi^2} \text{ or } \boxed{r = \sqrt[3]{GM\frac{T^2}{4\pi^2}}}$$

Use this formula to calculate the radius of a geostationary orbit ($T = 24\,\text{hr}$).

Slightly re-arranged this formula gives **Kepler's third law**: $\boxed{\frac{r^3}{T^2} = G\frac{M}{4\pi^2} = \text{constant}}$

We can also make T the subject:

$$T^2 = \frac{r^3 4\pi^2}{GM} \text{ or } \boxed{T = 2\pi\sqrt{\frac{r^3}{GM}}}$$