Physics Year 13 (Level 8)

Binding Energy and the release of energy from nuclear reactions (fission and fusion)

I. If you look at the rest mass of any nucleus and compare this with the sum of the rest masses of the individual nucleons you will see that the rest mass of the nucleus is always the *smaller* of the two. This is a **mass deficit**. With the equivalence between Energy and Mass expressed by $E = mc^2$ this mass deficit can be considered as an **energy deficit** of the nucleus. This amount of energy is needed to split the nucleus into individual nucleons. This energy deficit thus holds the nucleus together and is therefore called **Binding Energy**. But remember that it actually is an energy deficit, a shortage of (Potential) energy. If we divide the binding energy by the number of nucleons in the nucleus we get the Binding Energy per Nucleon (BEpN) (see the graph in the course book p.233).

Let us do some calculations to verify this.

Calculate the BEpN for the alpha-particle $^{4}_{2}\mathbf{He}$

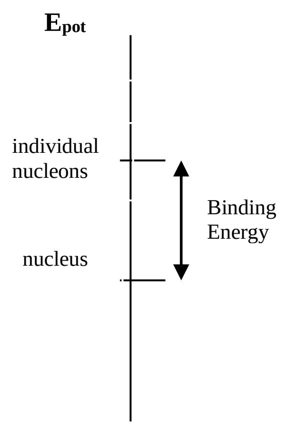
The restmass for the nucleus is 6.64591×10^{-27} kg
The restmass for the individual nucleons is

2 x $^{1}_{1}\mathbf{H}$ is 2 x 1.67338×10^{-27} =	3.34676×10^{-27} kg
2 x $^{1}_{0}\mathbf{n}$ is 2 x 1.67483×10^{-27} =	3.34966×10^{-27} kg
	6.69642×10^{-27} kg

This confirms that the restmass of the nucleus is smaller than the combined restmass of the nucleons.
The mass deficit is $6.69642 \times 10^{-27} - 6.64591 \times 10^{-27} = 0.05051 \times 10^{-27}$ kg.
This is equivalent to a Binding Energy of $E=mc^2 = 0.05051 \times 10^{-27} \times (2.998 \times 10^{8})^2 = 0.45398 \times 10^{-11}$ J.
The BEpN is thus $0.45398 \times 10^{-11} / 4 = 0.11350 \times 10^{-11}$ J. This is the value that is plotted along the vertical axis of the Binding Energy graph in your book.

Now also calculate the BEpN for $^{235}_{92}\mathbf{U}$ *and for* $^{142}_{56}\mathbf{Ba}$. *Compare the results with the graph in your book.*

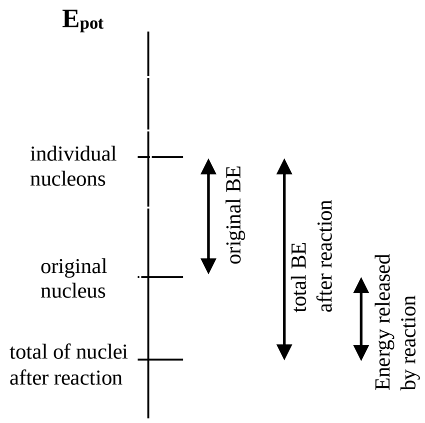
We can plot the Binding Energy as is done in the graph opposite. We plot Potential Energy vertically (positive upward). The individual nucleons combined have a higher Potential Energy than the nucleus. The difference is the Binding Energy.



II. If we have a nuclear reaction (Fission or Fusion) which produces nuclei which **go up** in the BEpN curve, the result of the reaction has a larger Binding Energy. Remember the total number of nucleons is the same before and after the reaction. It will require more energy to split those resulting nuclei into individual nucleons. Therefore this nuclear reaction will **produce** energy to make up for the difference (see graph opposite).

III. A **fission reaction** produces energy if the original (larger) nucleus is lower in the Binding Energy curve than the resulting (smaller) nuclei, i.e. anywhere to the **right of the culmination point** around mass number 50.

IV. A **fusion reaction** produces energy if the original (smaller) nuclei are lower in the Binding Energy curve than the resulting (larger) nucleus, i.e. anywhere to the **left of the culmination point**.



Let us confirm this with numerical examples of a typical fission and fusion reaction:

A. **FISSION**

$$^{239}_{94}\mathbf{Pu} + {}^{1}_{0}\mathbf{n} \rightarrow {}^{93}_{38}\mathbf{Sr} + {}^{142}_{56}\mathbf{Ba} + 5\ {}^{1}_{0}\mathbf{n}$$

Rest mass before:	$^{239}_{94}$**Pu**	396.92935 x 10^{-27}
	$^{1}_{0}$**n**	1.67483 x 10^{-27}
	Total	398.60418 x 10^{-27} kg
Rest mass after:	$^{93}_{38}$**Sr**	154.27837 x 10^{-27}
	$^{142}_{56}$**Ba**	235.64216 x 10^{-27}
	5 $^{1}_{0}$**n**	8.29468 x 10^{-27}
	Total	398.29468 x 10^{-27} kg

So the **mass decrease** is: $\Delta m = 0.3095 \times 10^{-27}$ kg. This relates to an **energy decrease** of $E = \Delta m\, c^2 = 0.3095 \times 10^{-27} \times (2.998 \times 10^8)^2 = 2.782 \times 10^{-11}$ J. Because of conservation of energy the difference in energy must be compensated by the release of that same amount of energy, i.e. this reaction **produces** 2.782 x 10^{-11} J.

B. **FUSION**

$$^{2}_{1}\mathbf{H} + {}^{3}_{1}\mathbf{H} \rightarrow {}^{4}_{2}\mathbf{He} + {}^{1}_{0}\mathbf{n}$$

Rest mass before:	$^{2}_{1}$**H**	3.34330 x 10^{-27}
	$^{3}_{1}$**H**	5.00784 x 10^{-27}
	Total	8.35114 x 10^{-27} kg
Rest mass after:	$^{4}_{2}$**He**	6.64591 x 10^{-27}
	$^{1}_{0}$**n**	1.67483 x 10^{-27}
	Total	8.32074 x 10^{-27} kg

So the **mass decrease** is: $\Delta m = 0.03040 \times 10^{-27}$ which relates to $0.03040 \times 10^{-27} \times (2.998 \times 10^8)^2 = 0.2732 \times 10^{-11}$ J **decrease in energy**. This is the amount of energy **produced** by this reaction.