Calculus Year 13 (Level 8)

Summary 1

Algebra

Factorisation

COMMON FACTOR $\frac{3x^2}{4y}+\frac{x}{2y}=\frac{x}{2y}\left(\frac{3x}{2}+1\right)$

GROUPING $2x-6+xy-3y=2(x-3)+y(x-3)=(x-3)(2+y)$

QUADRATICS

Coefficient of x is 1 $x^2-2x-15=(x-5)(x+3)$ (sum=-2; product=-15)

Coefficient of x is not 1 $3x^2-10x-8$ Find a and b such that $a\times b=3x^2\times(-8)=-24x^2$ and $a+b=-10x$. Solution: $a=-12x$ and $b=2x$. Re-write the expression as: $3x^2-12x+2x-8$ and find common factors: $=3x(x-4)+2(x-4)=(3x+2)(x-4)$

DIFFERENCE OF SQUARES General rule: $a^2-b^2=(a+b)(a-b)$

$$1-9x^2=1^2-(3x)^2=(1+3x)(1-3x)$$

Solving Quadratic Equations

BY FACTORISATION $x^2-x-6=0 \rightarrow (x-3)(x+2)=0$ Hence $x_{1,2}=3,-2$

BY FORMULA $ax^2+bx+c=0$ has roots $x_{1,2}=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

DISCRIMINANT $\Delta=b^2-4ac$

$> 0 \rightarrow$ two real solutions

$= 0 \rightarrow$ one repeated solution

$< 0 \rightarrow$ two imaginary solutions

COMPLETING THE SQUARE *Square of half the coefficient of x completes the square*

$$x^2-6x+3=0\rightarrow x^2-6x+9-6=0\rightarrow$$

$$(x-3)^2-6=0\rightarrow x-3=\pm\sqrt{6}\rightarrow x_{1,2}=3\pm\sqrt{6}$$

Quadratic Inequalities

$x^2-4x+3>0$ Factorise $(x-1)(x-3)>0$

Hence function = 0 at $x_{1,2}=1,3$ Then just try whether inequality is true inside or outside the interval (1,3)

Indices and Surds

RULES FOR POWERS: $a^p \times a^q = a^{(p+q)}$ $\qquad \frac{1}{a^q} = a^{-q}$ hence $\frac{a^p}{a^q} = a^{(p-q)}$

$(a^m)^n = a^{(mn)}$ $\qquad a^0 = 1$ $\qquad \sqrt{a} = a^{1/2}$ $\qquad \sqrt[m]{a^n} = a^{n/m}$

SIMPLIFYING SURD EXPRESSION Find common square factors $\sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2}$

Multiply with *conjugate* of denominator

$$\frac{3-\sqrt{2}}{2-\sqrt{3}} = \frac{(3-\sqrt{2})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})} = \frac{(3-\sqrt{2})(2+\sqrt{3})}{4-3} = 6-2\sqrt{2}+3\sqrt{3}-\sqrt{6}$$

SOLVING SURD EQUATIONS Remove radicals by squaring both sides.

But check for extraneous roots

$\sqrt{2x+15} - \sqrt{x+4} = 2$ or $\sqrt{2x+15} = 2 + \sqrt{x+4}$

Square: $2x+15 = 4 + 4\sqrt{x+4} + x + 4$ or $4\sqrt{x+4} = x+7$

Now square again $16(x+4) = x^2 + 14x + 49$ or $x^2 - 2x - 15 = 0$ or

$(x-5)(x+3) = 0$ and $x_{1,2} = 5, -3$

Check these roots in the original equation.

Equations and Inequations

General revision. See book Ex. 1.16

Remember to reverse the sign when multiplying inequalities with negative numbers

Simplify Algebraic Fractions

COMMON FACTOR IN NUMERATOR AND DENOMINATOR

$$\frac{x^3-4x}{x^2-6x-16} = \frac{x(x^2-4)}{(x-8)(x+2)} = \frac{x(x-2)(x+2)}{(x-8)(x+2)} = \frac{x(x-2)}{(x-8)}$$

CREATE COMMON DENOMINATOR

$$\frac{2x}{a^2b} - \frac{3x}{ab^2} = \frac{2bx}{a^2b^2} - \frac{3ax}{a^2b^2} = \frac{x(2b-3a)}{a^2b^2}$$

Remainder - Factor Theorems

REMAINDER THEOREM Divide $p(x)$ by $x-a$ then remainder is $p(a)$

$\frac{x^2-3x+5}{x+2} \rightarrow$ hence $a = -2$ Remainder is $p(-2) = 4+6+5 = 15$

Similarly divide $p(x)$ by $(ax-b)$ then remainder is $p\left(\frac{b}{a}\right)$

FACTOR THEOREM Used to factorise Polynomials

Find numbers for which $p(a) = 0$ then $(x-a)$ is a factor of the polynomial

$x^3 - 5x^2 + 2x + 8$ Try all factors of 8 i.e. $\pm 1, \pm 2, \pm 4, \pm 8$ and check if $p(a) = 0$

This works for $x = -1, +2, +4$ hence the factors are $(x+1)(x-2)(x-4)$

Changing Subject of an Equation

General revision. See book Ex. 2.5

Functions

The function $y = f(x)$ **maps** the set of *x* **onto** the set of *y.*

The set of *x* is the **domain**; the set of *y* is the **range** of the function

Any value of x can only have one value of y associated with it,
otherwise $f(x)$ is not a function

Notations: $4x+5$ $y = 4x+5$ $f(x) = 4x+5$

$f : x \mapsto 4x+5$ (mapping) $\{(x, y) : y = 4x+5\}$ (set builder)

INVERSE FUNCTION swap *x* and *y* in $y = f(x)$

Ex: $y = f(x) = x+6$ $f^{-1} \rightarrow x = y+6$ or $y = x-6$ Hence $f^{-1}(x) = x-6$

COMPOSITE FUNCTION $f \circ f^{-1} \equiv x$

Ex: substitute $y = x-6$ for x in $y = x+6$ i.e. $y = (x-6)+6 = x$